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Dependence of non-classical behavior of OPI Hamiltonian on the strength of coherence of initial light

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Abstract . Nonclassical properties of optical parametric interaction Hamiltonian (OPI), such as appearance of squeezed states, have been investigated *via* density matrix formalism where the partially coherent lights with Laguerre or Laser distributions have been chosen as initial states It is shown that the nonclassical behavior of light drastically depends on strength of coherence of initial lights

Keywords : Light squeezing, optical parametric interaction, density matrin formalism

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1. Introduction

The study of nonlinear process in the quantum optics domain has led to the observation of some quantum phenomena, whose observation in classical physics is not possible, even though these quantum phenomena can play very important role in Optical Integrated Circuits (OIC). One of the most important effects is Optical Parametric Interactions (OPI). The basic configuration involves an input signal with frequency ω_1 incident on a nonlinear crystal together with a pump wave at frequency ω_3, where $\omega_3 > \omega_1$ [1, 4, 5]. The amplification of the ω_1 wave is accompanied by a generation of an idler wave at $\omega_2 = \omega_3 - \omega_1$. The importance of this phenomenon is due to its ability to convert the output power of a laser pump to a coherent and idler frequency output signals where the latter can be tuned continuously over a wide range [1, 6, 7]. This nonlinear effect acts as a phase sensitive amplifier [1, 8], and this sensitivity is at the heart of the recent noise squeezing experiments [1]. Parametric interactions between different modes have been already studied enough and the most common case is the Hamiltonian which is a quadratic function of annihilation and creation operators, where the bilinear Hamiltonian

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can be obtained from a trilinear one by treating the pump semiclassically. Although this analysis has been successful for some applications, it obviously misses a variety of other possibilities [3]. For example, it has been shown that the transmission of a weak pump field from a high quality cavity containing a $\chi^{(2)}$ material exhibits new types of resonant structures if the interaction is strong [3]. Clearly it is important to study the full structure of energy levels of the OPI Hamiltonian and to calculate other observables. Some early work in this direction has been done by Perina *et al* [3], Kupiszewski and Rzazewski [2] and many others, but in all these works only special cases have been considered, for example, the pump has been treated semiclassically as the frequencies of signal and idler waves have been taken the same, also the authors in [10] have studied OPI Hamiltonian in general case with coherent states as initial states. While in this paper choosing the partially coherent light as an initial state, the nonclassical behavior of OPI has been studied *via* density matrix formalism. In order to take account the effect of thermal noises in laser sources, we have considered the Laguerre or Laser distribution. By tuning the parameters S and N one can rather smoothly switch between thermal (Bose-Einstein) and pure coherent states [8-10].

The paper is organized as : Modeling and formulation of OPI are presented in Section 2. In Section 3, we discuss the nonclassical behavior of OPI process using the Density Matrix formalism for different initial states. In Section 4 we obtain the average of signal photon number. The paper is ended with a conclusion.

2. OPI Hamiltonian and its solution

In this section, we will present the method of solving OPI Hamiltonian which is given by eq.(2.1)

$$H = \hbar\omega_1 a_1^\dagger a_1 + \hbar\omega_2 a_2^\dagger a_2 + \hbar\omega_3 a_3^\dagger a_3 - h(a_1^\dagger a_2^\dagger a_3 + a_1 a_2 a_3^\dagger), \tag{2.1}$$

where h is the interaction coefficient. It is straightforward to show that this Hamiltonian possesses three constants of motion, namely :

$$\hat{k} = a_1^\dagger a_1 + a_3^\dagger a_3 ,$$

$$\hat{p} = a_2^\dagger a_2 + a_3^\dagger a_3 ,$$

$$\hat{I} = a_1^\dagger a_2^\dagger a_3 + a_1 a_2 a_3^\dagger . \tag{2.2}$$

In order to solve this Hamiltonian, one must choose some suitable base vectors. Therefore, if we choose the simultaneous eigenvectors of the first two constants of motion given in eq. (2.2) as the base vectors, then obviously the OPI Hamiltonian would have block diagonal representation in this basis. Hence in order to obtain the spectrum of Hamiltonian we just need to diagonalize these blocks of block diagonal Hamiltonian. Actually blocks of up to 9×9 can be diagonalized analytically while the rest must be diagonalized numerically. Taking the eigenvalues of the operators $\hat{k}$ and $\hat{p}$, *i.e.* k and p, as the first two constants of motion, then their simultaneous eigendirections for given values of k and p are :

$$|k, p, i> = |k-i>|p-i>|i> \tag{2.3}$$

where i takes values between zero and the minimum of the k and p. First, we give the matrix elements of $H(H_{ij})$ in terms of the basis given by eq. (2.2).

$$< j, p-j, k-j \mid H \mid k-i, p-i, i > = (\hbar\omega_1(k-i) + \hbar\omega_2(p-i)$$

$$+ \hbar\omega_3(k-i))\delta_{i,j} - h(\sqrt{(k-i)(p-i)(i+1)})\delta_{i+1,j}$$

$$-h(\sqrt{(k-i+1)(p-i+1)(i-1)})\delta_{i-1,j} \, . \tag{2.4}$$

It is clear from eq. (2.4) that the matrix representation of H has nonvanishing matrix elements only along the diagonals just below or above the main diagonal. These nonvanishing matrix elements are :

$$H_{ii} = k\hbar\omega_1 + p\hbar\omega_2 \, , \tag{2.5}$$

$$H_{ii+1} = H_{i-1i} = h\sqrt{(k-i+1)(p-i+1)(i)} \, , \tag{2.6}$$

$$\omega_1 + \omega_2 = \omega_3 . \tag{2.7}$$

Using these matrix elements, we can find the eigenvectors and eigenvalues of Hamiltonian for given k and p denoted as ϕ_n^{kp} and E_n^{kp}, respectively. As an example, for $k = 2$ and $p \geq 2$, we come across a 2×2 block with eigenvalues

$$E_1^{k,p} = (2h\sqrt{kp} - \hbar(-2k\omega_1 - 2\omega_2 p))/2 \, ,$$

$$E_2^{k,p} = (-2h\sqrt{kp} - \hbar(-2k\omega_1 - 2\omega_2 p))/2$$

and eigenfunctions

$$\phi_1^{k,p} = (-((kp)^{1/2}/(k^{1/2}p^{1/2})), 1)/\sqrt{2} \, ,$$

$$\phi_2^{k,p} = ((kp)^{1/2}/(k^{1/2}p^{1/2}), 1)/\sqrt{2} \, , \tag{2.8}$$

Similarly, for the case of $k = 3$ and $p \geq 3$, and $k = 4$ and $p \geq 4$ eigenvalues and eigenvectors analytically can be calculated.

The blocks with $9 \leq min\ (k, p) \leq \infty$ can be diagonalized only numerically. As an example, we have given the results of k = 15th block in Table 1.

As already mentioned, the set of $\{ \mid \phi_n^{k,p} > \}$ are suitable complete basis for expressing any interesting quantity in the OPI process. Thus we will use this basis to express the matrix elements of density operator. Before closing this section we must point out that eigenvectors $\mid \phi_n^{k,p} >$ can be expressed in the chosen basis given in eq. (2.3) as :

$$\mid \phi_n^{k,p} > = \sum_{m=0}^{M} C_m^{n,k,p} \mid k-m, p-m, m > , \tag{2.9}$$

where $m = 0, 1, \ldots\ldots\ldots, M$ with M as the minimum of k, p ($M = \min(k, p)$). $C_m^{n,k,p}$ is the (m, n)-th element of the eigenvectors matrix.

3. Study of nonclassical behavior of OPI using density operator

In order to observe the squeezed state in OPI phenomenon, it is rather convenient to try to see it in combined fields. This is due to the fact that squeezed states are not conveniently generated

Table 1. ϕ_n^{kp} and E_n^{kp} for $k, p = 15$.

k,p	M			
15	16	10^{-18}	0.042110^{-15}	0.2939 0.3391 0.0739 −0.2414 −0.2435 0.0642 0.2898 0.1447
				−0.1943 −0.3161 −0.0733 0.2851 0.4439 0.3453 0.1571 0.0352
			0.047910^{-15}	0.3319 0.2539 −0.1043 −0.2764 −0.0343 0.2537 0.1533 −0 1871
				−0.2563 0.0616 0.3209 0.1516 −0.2585 −0.4684 −0.3447 −0.1163
			0.053110^{-15}	−0.3397 −0.1418 0.2125 0.1837 −0.1515 −0.2178 0.0964 0 2494
				−0.0325 −0.2784 −0.0559 0.2975 0.2013 −0.2645 −0.5123 −0 3170
			0 057510^{-15}	0.3160 0.0397 −0.2356 −0.0546 0.2168 0.0698 −0.2123 −0 0886
				0.2161 0.1149 −0.2278 −0 1573 0.2512 0.2452 −0.3014 −0 6202
			0.061310^{-15}	−0.3160 0 0397 0.2356 −0.546 −0.2168 0.0698 0.2123 −0 0886
				−0.2161 0.1149 0.2278 −0 1573 −0 2512 0.2452 0.3014 −0.6202
			0 065610^{-15}	0.3397 −0.1418 −0.2125 0.1837 0.1515 −0.2178 −0.0964 0.2498
				0.0325 −0.2784 0.0559 0.2975 −0.2013 −0.2645 0 5123 −0 3170
			0 070910^{-15}	−0 3319 0.2539 0.1043 −0.2764 0 0343 0 2537 −0 1533 −0.1871
				0.2563 0.0616 −0.3209 0.1516 0.2585 −0.4684 0 3447 −0 1163
			0.035710^{-15}	0 2323 0.3665 0.2620 −0 0469 −0.2921 −0 2352 0.0660 0.3076
				0.2522 −0.0472 −0 3380 −0.4283 −0 3270 −0.1666 −0 0541−0 0089
			0 076710^{-15}	−0.2939 0 3391 −0.0739 −0 2414 0 2435 0.0642 −0 2898 0 1447
				0 1943 −0.3161 0 0733 0.2851 −0.4439 0 3453 −0.1571 0 0352
			0 028910^{-}	0.1588 0.3227 0.3763 0 2256 −0.0666 −0.3026 −0 3097 −0.0850
				0.2114 0.3983 0 4059 0.2901 0.1524 0.0576 0.0144 0 0018
			0.083010^{-}	−0.2323 0.3665 −0.2620 −0 0469 0.2921 −0.2352 −0 0660 0 3076
				0 2522 −0.0472 0.3380 −0.4283 0.3270 −0 1666 0.0541 −0.0089
			0.089910	0.1588 −0.3227 0.3763 −0 2256 −0 0666 0 3026 −0.3097 0 0850
				0.2114 −0.3983 0.4059 −0.2901 0.1524 −0 0576 0 0144 −0 0018
			0.021710^{-}	0.0884 0.2223 0.3564 0 4013 0.2962 0.0626 −0.2012 −0 3829
				−0.4264 −0.3531 −0.2298 −0.1186 −0.0477 −0.0142 −0.0029 −0 0003
			0 097110^{-}	0.0884 −0.2223 0.3564 −0.4013 0.2962 −0.0626 −0.2012 0 3829
				0.4264 0.3531 −0.2298 0.1186 −0.0477 0.0142 −0.0029 0 0003
			0.014010	0.0347 0.1048 0.2138 0.3382 0.4383 0.4779 0.4443 0.3543
				0.2423 0.1413 0 0695 0.0282 0.0092 0.0022 0.0004 0 0000
			0.104710	−0.0347 0.1047 −0.2138 0.3382 −0.4383 0.4779 −0.4443 0.3543
				−0.2423 0.1413 −0.0695 0.0282 −0.0092 0.0022 −0.0004 0.0000

in terms of single fields. While due to strong field coupling, these states can be easily seen in the combined case. We define $x_{1,2}$ as the sum of fields x_1 and x_2 :

$$x_{1,2} = x_1 + x_2$$

together with

$$x_{1,2}^2 = x_1^2 + x_2^2 + 2x_1x_2 ,$$

where

$$x_1 = a_1 + a_1^\dagger$$

$$x_2 = a_2 + a_2^\dagger$$

and

$$x_1^2 = a_1^2 + a_1^{\dagger 2} + a_1^\dagger a_1 + a_1 a_1^\dagger ,$$

$$x_2^2 = a_2^2 + a_2^{\dagger 2} + a_2^\dagger a_2 + a_2 a_2^\dagger .$$

Thus the standard deviation of combined fields

$$< (\Delta x_{12})^2 > = < x_{12}^2 > - < x_{12} >^2$$

or

$$< (\Delta x_{12})^2 > = < \Delta x_1^2 > + < \Delta x_2^2 > + 2(< x_1 x_2 > - < x_1 > < x_2 >) . \qquad (3.1)$$

can be expressed as a function of time, and the parameters of the OPI, and the initial states if we calculate $< x_1 >$, $< x_1^2 >$, $< x_2 >$, $< x_2^2 >$, and $< x_1 x_2 >$ respectively. Here, these averages have been calculated *via* density matrix formalism where the laser or laguerre distribution is choosen as an initial states. Solving the time evolution of $\rho(t)$ for a given Hamiltonian without any explicit time independence, with corresponding equation of motion given in eq. (3.2).

$$\frac{\partial \rho}{\partial t} = \frac{1}{i\hbar}[H, \rho] , \qquad (3.2)$$

we obtain

$$\rho(t) = e^{iHt/\hbar} \rho(0)\, e^{-iHt/\hbar} \qquad (3.3)$$

with ρ (0) as the initial density matrix. In practice, we can hardly have a pure coherent state but rather we have partial coherent states. So for $\rho(0)$, we choose laser distribution which is very suitable experimentally, because it is a combination of thermal and pure coherent states. Actually the density operator in general, can be written as

$$\rho(t) = \int \frac{d^2\alpha}{\pi} \phi(\alpha) | \alpha >< \alpha | , \qquad (3.4)$$

where $\phi(\alpha)$ is weight function with normalization

$$\int \phi(\alpha) | \alpha >< \alpha | \frac{d^2\alpha}{\pi} = 1 , \qquad (3.5)$$

which follows from the normalization of ρ, and $|\alpha\rangle$ is coherent state ket. Here in this work, the displaced Gaussian function has been adopted as a weight function, that is we have

$$\phi(\alpha) = \frac{e^{-|\beta-\alpha|^2/(N/k)}}{(N/k)^k} \tag{3.6}$$

with $|\beta|^2 = S$ as a signal strength and N as a noise strength, respectively. In order to study the nonclassical behavior of O.P.I., *i.e.* to calculate the average quantities defined in eq. (3.1), we must calculate the matrix elements of density operators in a suitable basis. Therefore, we choose the eigenkets of Hamiltonian described by eq. (2.3). In this basis, the matrix elements are :

$$\rho_{n',k',p',n,k,p} = \langle \phi_{n'}^{k',p'} | \rho(t) | \phi_n^{k,p} \rangle$$

$$= e^{i(E_n^{k,p} - E_{n'}^{k',p'})t/\hbar} \sum_{m,m'=0}^{M,M'} C_{m'}^{n',k',p'} C_m^{n,k,p} \langle m', p'-m', k'-m' | k-m, p-m, m \rangle$$

$$= e^{i(E_n^{k,p} - E_{n'}^{k',p'})t/\hbar} \sum_{m,m'=0}^{M,M'} C_{m'}^{n',k',p'} C_m^{n,k,p} \int \frac{d^2\alpha_{1,2,3}}{\pi} \phi(\alpha_{1,2,3}) \times$$

$$\langle m', p'-m', k'-m' | k-m, p-m, m \rangle . \tag{3.7}$$

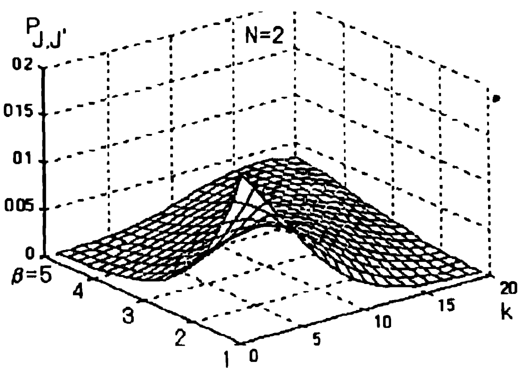

Figure 1 Coefficient of distribution *vs.* k and β (Noise level = 2).

Using the following integral representation of Bessel function and Laguerre polynomials

$$\int_{\phi=0}^{2\pi} e^{(in\phi - x\sin(\phi))} d\phi = J_n(x) \tag{3.8}$$

and

$$n!\, L_n^{\alpha}(x) = e^x x^{-\frac{1}{2}\alpha} \int_{t=0}^{\infty} e^{-t} t^{n+\frac{\alpha}{2}} J_\alpha(2(xt)^{\frac{1}{2}}) dt \tag{3.9}$$

respectively, the density matrix elements $\rho_{n',n}$ given in eq. (3.5) take the following form

$$\rho^{k,p,k',p'}_{n,n'} = \sum_{m,m'=0}^{M,M'} e^{i(E^{k,p}_{n} - E^{k',p'}_{n'})t/\hbar} C^{n',k',p'}_{m'} \; C^{n,k,p}_{m} \times P^{k-m}_{k'-m'} P^{p-m}_{p'-m'} P^{m}_{m'}, \tag{3.10}$$

where $P_{J,J'}$ is defined as :

$$P_{J,J'} = (-1)^{PWL} \sqrt{\frac{min(J,J')!}{max(J,J')!}} \; {}^{\frac{|\beta|^2}{1+N/k}} \; \frac{|\beta|^{PWL}}{(N/k)^{k+PWL}(1+N/k)^{max(J,J')+1}}$$

$$\times L^{PWL}_{min(J,J')}\left(\frac{-|\beta|^2}{N/k(1+N/k)}\right). \tag{3.11}$$

As it is shown in Figures 1 and 2, the coefficient distribution $P_{J,J'}$ is almost negligible for higher values k and β.

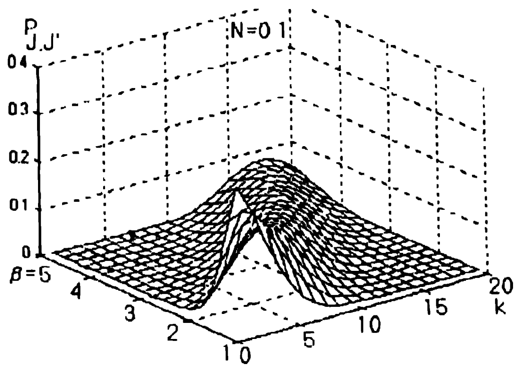

Figure 2 Coefficient of distribution *vs* k and β (Noise level = 0.1)

Here, $PWL = max\,(J,J') - min\,(J,J')$ and $L^{\alpha}_{n}(x)$ is generalized laguerre polynomial. In the calculation of combined standard deviation, we need to evaluate $<x_{1,2}>$ and $<(x_{1,2})^2>$. Using the density matrix elements given in eq. (3.3), the $<x_{1,2}>$ and $<(x_{1,2})^2>$ take the following form

$$< x_{1,2} > = \mathrm{Tr}\,(\rho x_{1,2})\; x_{1,2}{}^{k,p,k',p'}$$

$$= \sum_{m=0}^{M} C^{n',k',p'}_{m} \; C^{n,k,p}_{m} \times [\sqrt{k'-m}\,\delta_{p,p'}\delta_{k,k'-1} + \sqrt{k'-m+1}\,\delta_{p,p'}\delta_{k,k'+1} +$$

$$\sqrt{p'-m}\,\delta_{k,k'}\delta_{p,p'-1} + \sqrt{p'-m+1}\,\delta_{k,k'}\delta_{p,p'+1}] \tag{3.12}$$

and

$$<(x_{1,2})^2> = \mathrm{Tr}\,(\rho(x_{1,2})^2)$$

$$x_{1,2^2_{n,n'}}{}^{k,p,k',p'} = \sum_{m=0}^{M} C^{n',k',p'}_{m} C^{n,k,p}_{m} \times [2(1+k'+p'-2m')\delta_{p,p'}\delta_{k,k'}$$

$$+\sqrt{(k'-m)(k'-m-1)}\delta_{p,p'}\delta_{k,k'-2}+\sqrt{(k'-m+1)(k'-m+2)}\delta_{p,p'}\delta_{k,k'+2}$$

$$+\sqrt{(p'-m)(p'-m-1)}\delta_{k,k'}\delta_{p,p'-2}+\sqrt{(p'-m+1)(p'-m+2)}\delta_{k,k'}\delta_{p,p'+2}$$

$$+2\sqrt{(k'-m)(p'-m)}\delta_{k,k'-1}\delta_{p,p'-1}+2\sqrt{(k'-m)(p'-m+1)}\delta_{k,k'-1}\delta_{p,p'+1}$$

$$+2\sqrt{(k'-m+1)(p'-m)}\delta_{k,k'+1}\delta_{p,p'-1}+2\sqrt{(k'-m+1)(p'-m+1)}\delta_{k,k'+1}\delta_{p,p'+1}] \quad (3.13)$$

respectively. Since our density matrix is not normalized we also need to calculate $\mathrm{Tr}(\rho)$ which is

$$\mathrm{Tr}(\rho)=\sum_{p=0}\rho_{0,0}^{0,p,0,p}+\sum_{k=1}\rho_{0,0}^{k,0,k,0}+$$

$$\sum_{k,p=1}^{\infty}\sum_{n,n'=0}^{M,M'}\sum_{m1,m2,m3=0}^{M1,M2,M3}[C_{m1}^{n,k,p}C_{m2}^{n',k,p}C_{m3}^{n,k,p}C_{m3}^{n',k,p}$$

$$\times e^{i(E_n^{k,p}-E_{n'}^{k,p})t/\hbar}P_{k-m2,\,k-m1}\;P_{p-m2,\,p-m1}\;P_{m2,\,m1}\,. \quad (3.14)$$

Now, truncating over infinite sums by simply taking k and p smaller than some given numbers, which in our case are below 14 and 15, these quantities can be calculated numerically for given values of the parameters β_1, n_1, β_2, n_2, and β_3, n_3 as a function of time. Hence, one can calculate numerically $<(\Delta x_{1,2})^2>$ as a function of time. The result is shown in Figure 3. As we see, there is no trace of squeezed state in single field cases, for any intervals of time. We obtain that, the squeezed states are generated in the combined fields in the time interval $[0, 3\times 10^{-17}]$.

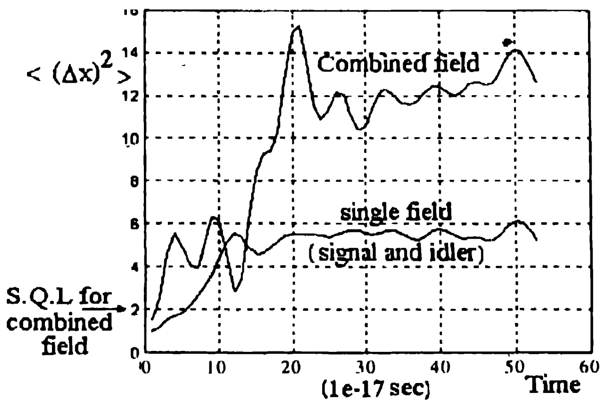

Figure 3. Dispersion of single and combined fields ($z_1 = z_2 = 1.2$, $z_3 = 1.5$ and all phase = 0) (Noise = 0).

4. The average of signal photon number

In this section, we will illustrate the special behavior of the average signal photon number in short and long time for degenerate parametric interactions. According to density operator approach developed in Section 3, the average of signal photon number $<N_1>$ can be given as :

$$<N_1>=\mathrm{Tr}\,(\rho N_1)\,,$$

where

$$\rho_{n',n}^{k,k'} = \sum_{m,m'=0}^{M,M'} C_m^{n,k} C_{m'}^{n',k'} \, e^{i(E_n^k - E_{n'}^{k'})t/\hbar} P_{k'-2m',k-2m} \, P_{m',n}$$

and

$$N_{1\,n,n'}^{\,k,k'} = \sum_{m=0}^{M} C_m^{n,k} C_{m'}^{n',k'} (k' - 2m')\delta_{m,m'}\delta_{k,k'} \,;$$

then

$$< N_1 > = \frac{1}{Tr(\rho)} [\rho_{0,0}^2 + \sum_{k=2}^{\infty} \sum_{n,n'=0}^{M,M'} \sum_{m1,m2,m3=0}^{M1,M2,M3} \times$$

$$C_{m1}^{n,k} C_{m2}^{n',k} C_{m3}^{n,k} C_{m3}^{n',k} \, (k - 2m3) \, e^{i(E_n^k - E_{n'}^k)t/\hbar} P_{k-m',k-m} P_{m',m} \tag{4.1}$$

The short time and also long time behaviors of $<N_1>$, as well as the average number of signal photon given in eq. (4.1), are illustrated in Figures 4 and 5, respectively. Comparing the

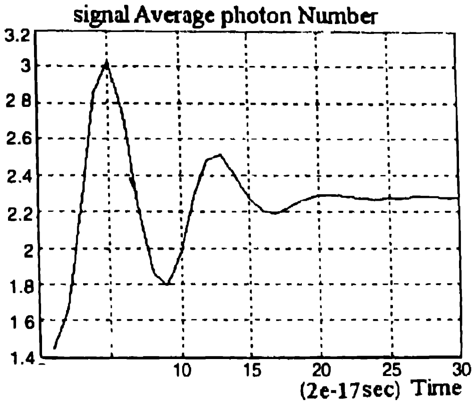

Figure 4 Average signal photon number vs. to time (short time) ($z_1 = z_2 = 1.2$, $z_3 = 1.5$ and all phases = 0) (Noise = 0)

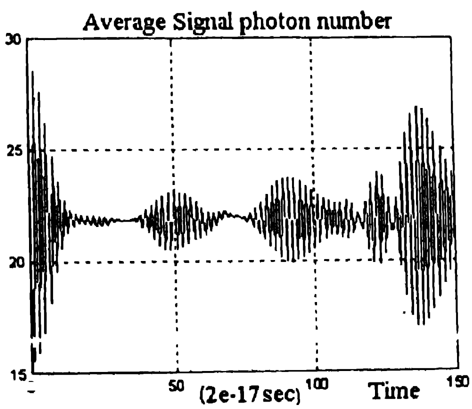

Figure 5. Average signal photon number *vs* time ($z_1 = z_2 = z_3 = 5$ and all phases = 0) (Noise = 0).

Figures 4 and 5 with Figure 3, we see that fluctuation of $<N_1>$ and also the combined squeezed state appear in the same time interval. This is due to the fact that the interaction is the most important factor for the observation of combined squeezed states and fluctuation of $<N_1>$. Also in Figure 6, we have shown the dependence of $< x_{1,2}^2 >$ in the most squeezed state on the parameters of $\rho(0)$, that is noise strength N and signal amplitude β. We see that the maximum squeezing occurs in the initial pure coherent state which is in agreement with the result of [10]. Note that the squeezing decreases with increasing of noise strength N and it disappears in the case of verv larøe N, that is, initial thermal cases.

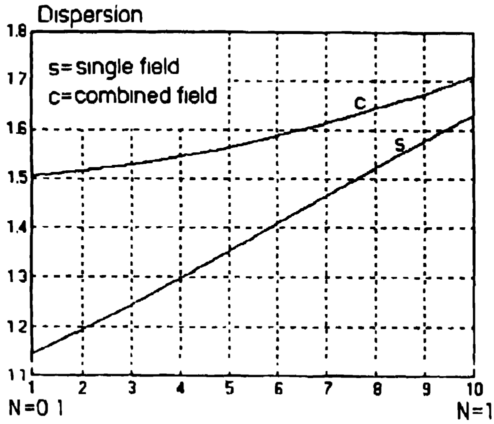

Figure 6. Dispersion of combined and single fields *vs.* noise level (k = 1 and β = 1 2)

5. Conclusion

Here, a tractable method of solving OPI Hamiltonian in the Schroedinger picture has been developed which is very feasible for computer simulation and also gives a visual conception from its interacting dynamics *via* density matrix formalism using Laser and Laguerre distributions as initial states. The obtained result for different values of initial state parameters are in complete agreement with the already existing ones, which are calculated by other methods such as Wigner distribution and truncation of infinite coupled equations. The approach is also applicable for the case of OPI with travelling waves or higher order interacting waves, either in pure or mixed states. Again in the latter case, one deals with the density matrix rather than a wave function.

Appendix

Here, we give the eigenvalues $E_n^{k,p}$ corresponding to 4 × 4 and 5 × 5 blocks of OPI Hamiltonian :

$$E_1^{k,p} = (-(\hbar(-2k\omega_1 - 2\omega_2 p)) + 2h(7 - 4k - 4p + 3kp +$$

$$(49 - 56k + 16k^2 - 56p + 62kp - 18k^2 p + 16p^2 - 18kp^2 + 6k^2 p^2)^{1/2})^{1/2})/2 ,$$

$$E_2^{k,p} = (-(\hbar(-2k\omega_1 - 2\omega_2 p)) - 2h(7 - 4k - 4p + 3kp +$$

$$(49 - 56k + 16k^2 - 56p + 62kp - 18k^2 p + 16p^2 - 18kp^2 + 6k^2 p^2)^{1/2})^{1/2})/2 ,$$

$$E_3^{k,p} = (-(\hbar(-2k\omega_1 - 2\omega_2 p)) + 2h(7 - 4k - 4p + 3kp -$$

$$(49 - 56k + 16k^2 - 56p + 62kp - 18k^2 p + 16p^2 - 18kp^2 + 6k^2 p^2)^{1/2})^{1/2})/2,$$

$$E_4^{k,p} = (-(\hbar(-2k\omega_1 - 2\omega_2 p)) - 2h(7 - 4k - 4p + 3kp -$$

$$(49 - 56k + 16k^2 - 56p + 62kp - 18k^2 p + 16p^2 - 18kp^2 + 6k^2 p^2)^{1/2})^{1/2})/2,$$

$$E_1^{k,p} = \hbar(k\omega_1 + \omega_2 p),$$

$$E_2^{k,p} = (-(\hbar(-2k\omega_1 - 2\omega_2 p)) + 2h(25 - 10k - 10p + 5kp +$$

$$(553 - 404k + 76k^2 - 404p + 274kp - 50k^2 p + 76p^2 - 50kp^2 + 10k^2 p^2)^{1/2})^{1/2})/2,$$

$$E_3^{k,p} = (-(\hbar(-2k\omega_1 - 2\omega_2 p)) - 2h(25 - 10k - 10p + 5kp +$$

$$(553 - 404k + 76k^2 - 404p + 274kp - 50k^2 p + 76p^2 - 50kp^2 + 10k^2 p^2)^{1/2})^{1/2})/2,$$

$$E_4^{k,p} = (-(\hbar(-2k\omega_1 - 2\omega_2 p)) + 2h(25 - 10k - 10p + 5kp -$$

$$(553 - 404k + 76k^2 - 404p + 274kp - 50k^2 p + 76p^2 - 50kp^2 + 10k^2 p^2)^{1/2})^{1/2})/2,$$

$$E_5^{k,p} = (-(\hbar(-2k\omega_1 - 2\omega_2 p)) - 2h(25 - 10k - 10p + 5kp -$$

$$(553 - 404k + 76k^2 - 404p + 274kp - 50k^2 p + 76p^2 - 50kp^2 + 10k^2 p^2)^{1/2})^{1/2})/2$$

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